# APPENDIX TO ：INTEGER SUPERHARMONIC MATRICES ON THE $F$－LATTICE 

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The table below displays a Farey quadruple $\left(p_{0}, q_{0}, p_{1}, q_{1}\right)=\left(\mathcal{C}\left(p_{1}, q_{1}\right), p_{1}, q_{1}\right)$ and the Laplacian of the odd child＇s standard and alternate tile odometers．We only draw the Laplacian of $p_{0}$ since the Laplacian of any odd $\left(\frac{n}{d}\right)$ is the rotated Laplacian of even $\left(\frac{d-n}{d+n}\right)$ ． All quadruples with $14 \leq \operatorname{det}\left(L^{\prime}\left(p_{0}\right)\right) \leq 10^{4}$ are displayed．

| $\left(p_{0}, q_{0}, p_{1}, q_{1}\right)$ | standard tile odometer | alternate tile odometer |
| :---: | :---: | :---: |
| （1／2，1／3，0／1，1／1） | ＋ | 3 |
| $(2 / 3,3 / 5,1 / 2,1 / 1)$ | 4 | \％ |
| $(1 / 4,1 / 5,0 / 1,1 / 3)$ | Wh | \％ |
| $(3 / 4,5 / 7,2 / 3,1 / 1)$ | \％ | 参 |
| $(2 / 5,3 / 7,1 / 2,1 / 3)$ | 发 | 屈 |
| $(1 / 6,1 / 7,0 / 1,1 / 5)$ | 空极 | 等 |
| $(4 / 5,7 / 9,3 / 4,1 / 1)$ | \％ | 萝 |
| $(5 / 6,9 / 11,4 / 5,1 / 1)$ | \％ | 多 |
| $(2 / 7,3 / 11,1 / 4,1 / 3)$ |  | $x^{2 y^{2}}$ |
| $(1 / 8,1 / 9,0 / 1,1 / 7)$ |  | 翏 |
| $(4 / 7,5 / 9,1 / 2,3 / 5)$ | 家家 | 事安 |
| $(6 / 7,11 / 13,5 / 6,1 / 1)$ |  |  |
| （3／8，5／13， $2 / 5,1 / 3)$ |  |  |
| $(2 / 9,3 / 13,1 / 4,1 / 5)$ | $5 y^{2} \times 2 x^{2}$ | $5 x^{2}$ |
| （5／8，7／11，2／3，3／5） | 鹤 |  |
| $(1 / 10,1 / 11,0 / 1,1 / 9)$ | ＊－ | ，＜ |
| $(7 / 8,13 / 15,6 / 7,1 / 1)$ |  |  |
| $(4 / 9,5 / 11,1 / 2,3 / 7)$ |  |  |
| $(3 / 10,5 / 17,2 / 7,1 / 3)$ |  |  |



(4/19, $7 / 33,3 / 14,1 / 5)$
(13/16, 17/21, 4/5, 9/11)
(3/20, 5/33, 2/13, 1/7)
$(2 / 21,3 / 31,1 / 10,1 / 11)$
(7/18, 9/23, 2/5, 5/13)
(10/17, 17/29, 7/12, 3/5)
(12/17, 19/27, 7/10, 5/7)
(6/19, 11/35, 5/16, 1/3)
(14/17, 19/23, 5/6, 9/11)
(4/21, 7/37, 3/16, 1/5)
(11/18, 19/31, 8/13, 3/5)

$(8 / 19,13 / 31,5 / 12,3 / 7)$
(3/22, 5/37, 2/15, 1/7)
$(2 / 23,3 / 35,1 / 12,1 / 11)$
(13/18, 21/29, 8/11, 5/7)
(7/20, 13/37, 6/17, 1/3)
(10/19, 11/21, 1/2, 9/17)
(12/19, 17/27, 5/8, 7/11)
(9/20, 13/29, 4/9, 5/11)
(5/22, 7/31, 2/9, 3/13)
$(14 / 19,17 / 23,3 / 4,11 / 15)$
(4/23, 5/29, 1/6, 3/17)

(8/21, 11/29, 3/8, 5/13)
$(16 / 19,21 / 25,5 / 6,11 / 13)$
(11/20, 17/31, 6/11, 5/9)
$(2 / 25,3 / 37,1 / 12,1 / 13)$
(13/20, 15/23, 2/3, 11/17)
(10/21, 11/23, 1/2, 9/19)
(6/23, 7/27, 1/4, 5/19)
(5/24, 9/43, 4/19, 1/5)
(17/20, 23/27, 6/7, 11/13)
(9/22, 11/27, 2/5, 7/17)

$(4 / 25,5 / 31,1 / 6,3 / 19)$
(3/26, 5/43, 2/17, 1/9)
(16/21, 19/25, 3/4, 13/17)
(7/24, 9/31, 2/7, 5/17)
(13/22, 23/39, 10/17, 3/5)
(10/23, 17/39, 7/16, 3/7)
(6/25, 7/29, 1/4, 5/21)
(5/26, 9/47, 4/21, 1/5)
(15/22, 17/25, 2/3, 13/19)
$(4 / 27,7 / 47,3 / 20,1 / 7)$

(12/23, 13/25, 1/2, 11/21)
(17/22, 27/35, 10/13, 7/9)
(3/28, 5/47, 2/19, 1/9)
(14/23, 25/41, 11/18, 3/5)
(11/24, 17/37, 6/13, 5/11)
(7/26, 11/41, 4/15, 3/11)
$(16 / 23,23 / 33,7 / 10,9 / 13)$
(13/24, 19/35, 6/11, 7/13)

(5/28, 7/39, 2/11, 3/17)
$(4 / 29,7 / 51,3 / 22,1 / 7)$
$(12 / 25,13 / 27,1 / 2,11 / 23)$
(8/27, 11/37, 3/10, 5/17)
$(17 / 24,29 / 41,12 / 17,5 / 7)$
$(11 / 26,19 / 45,8 / 19,3 / 7)$
(19/24, 23/29, 4/5, 15/19)

$(6 / 29,11 / 53,5 / 24,1 / 5)$
(10/27, 13/35, 3/8, 7/19)
$(16 / 25,25 / 39,9 / 14,7 / 11)$
$(4 / 31,5 / 39,1 / 8,3 / 23)$
$(18 / 25,31 / 43,13 / 18,5 / 7)$
$(3 / 32,5 / 53,2 / 21,1 / 11)$
$(15 / 26,19 / 33,4 / 7,11 / 19)$


(11/28, 13/33, 2/5, 9/23)
(19/26, 27/37, 8/11, 11/15)
(5/32, 7/45, 2/13, 3/19)
(21/26, 25/31, 4/5, 17/21)
$(4 / 33,5 / 41,1 / 8,3 / 25)$


(11/30, 15/41, 4/11, 7/19)
$(17 / 28,31 / 51,14 / 23,3 / 5)$
(5/34, 9/61, 4/27, 1/7)
$(19 / 28,21 / 31,2 / 3,17 / 25)$
$(4 / 35,7 / 61,3 / 26,1 / 9)$



(20/29, 31/45, 11/16, 9/13)
(6/35, 7/41, 1/6, 5/29)
(17/30, 21/37, 4/7, 13/23)
$(14 / 31,23 / 51,9 / 20,5 / 11)$
$(22 / 29,25 / 33,3 / 4,19 / 25)$
(10/33, 13/43, 3/10, 7/23)
(24/29, 29/35, 5/6, 19/23)


15
(19/30, 31/49, 12/19, 7/11)
(13/32, 15/37, 2/5, 11/27)
(9/34, 13/49, 4/15, 5/19)
(8/35, 13/57, 5/22, 3/13)
$(23 / 30,33 / 43,10 / 13,13 / 17)$
$(18 / 31,25 / 43,7 / 12,11 / 19)$
(15/32, 23/49, 8/17, 7/15)


(24/31, 41/53, 17/22, 7/9)
(8/37, 11/51, 3/14, 5/23)
(7/38, 9/49, 2/11, 5/27)
(21/32, 23/35, 2/3, 19/29)
$(15 / 34,19 / 43,4 / 9,11 / 25)$
$(11 / 36,15 / 49,4 / 13,7 / 23)$


18
(4/41, 5/51, 1/10, 3/31)
(25/32, 43/55, 18/23, 7/9)
(10/37, 17/63, 7/26, 3/11)
(9/38, 13/55, 4/17, 5/21)
$(13 / 36,17 / 47,4 / 11,9 / 25)$

(19/34, 33/59, 14/25, 5/9)
(16/35, 27/59, 11/24, 5/11)
(7/40, 11/63, 4/23, 3/17)
(26/33, 37/47, 11/14, 15/19)
(21/34, 29/47, 8/13, 13/21)

(11/38, 13/45, 2/7, 9/31)
(5/42, 7/59, 2/17, 3/25)
(10/39, 11/43, 1/4, 9/35)
$(14 / 37,17 / 45,3 / 8,11 / 29)$
(17/36, 25/53, 8/17, 9/19)
$(25 / 34,39 / 53,14 / 19,11 / 15)$
(9/40, 11/49, 2/9, 7/31)

(27/34, 31/39, 4/5, 23/29)
(22/35, 27/43, 5/8, 17/27)

(16/37, 29/67, 13/30, 3/7)

(19/36, 29/55, 10/19, 9/17)
(24/35, 35/51, 11/16, 13/19)


(11/40, 19/69, 8/29, 3/11)
(26/35, 29/39, 3/4, 23/31)
(10/41, 11/45, 1/4, 9/37)
(14/39, 19/53, 5/14, 9/25)
(23/36, 39/61, 16/25, 7/11)


$(21 / 38,37 / 67,16 / 29,5 / 9)$
(10/43, 17/73, 7/30, 3/13)


(13/42, 17/55, 4/13, 9/29)
$(30 / 37,43 / 53,13 / 16,17 / 21)$



(23/40, 27/47, 4/7, 19/33)
(13/44, 21/71, 8/27, 5/17)
$(16 / 43,19 / 51,3 / 8,13 / 35)$
(19/42, 33/73, 14/31, 5/11)


(22/41, 37/69, 15/28, 7/13)
(11/46, 17/71, 6/25, 5/21)
(10/47, 13/61, 3/14, 7/33)
(18/43, 23/55, 5/12, 13/31)
(24/41, 31/53, 7/12, 17/29)


(29/40, 37/51, 8/11, 21/29)
$(14 / 45,19 / 61,5 / 16,9 / 29)$
(8/49, 9/55, 1/6, 7/43)
(26/41, 45/71, 19/30, 7/11)
(17/44, 29/75, 12/31, 5/13)



$(28 / 41,43 / 63,15 / 22,13 / 19)$
(5/52, 7/73, 2/21, 3/31)
(16/45, 21/59, 5/14, 11/31)
(11/48, 19/83, 8/35, 3/13)
(30/41, 49/67, 19/26, 11/15)



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(9 / 50,11 / 61,2 / 11,7 / 39)
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(14/47, 17/57, 3/10, 11/37)
(24/43, 43/77, 19/34, 5/9)
(21/44, 31/65, 10/21, 11/23)
(8/51, 13/83, 5/32, 3/19)


(29/42, 49/71, 20/29, 9/13)
(13/48, 23/85, 10/37, 3/11)
(7/52, 9/67, 2/15, 5/37)
(17/46, 27/73, 10/27, 7/19)


(31/42, 45/61, 14/19, 17/23)
(28/43, 41/63, 13/20, 15/23)
(11/50, 13/59, 2/9, 9/41)
(19/46, 31/75, 12/29, 7/17)


(25/44, 29/51, 4/7, 21/37)
(30/43, 37/53, 7/10, 23/33)
(9/52, 13/75, 4/23, 5/29)
(18/47, 31/81, 13/34, 5/13)
(27/44, 35/57, 8/13, 19/31)

(8/53, 11/73, 3/20, 5/33)
$(34 / 43,53 / 67,19 / 24,15 / 19)$
(7/54, 11/85, 4/31, 3/23)

(21/46, 37/81, 16/35, 5/11)

(17/48, 23/65, 6/17, 11/31)
$(26 / 45,41 / 71,15 / 26,11 / 19)$
(31/44, 43/61, 12/17, 19/27)
(11/52, 15/71, 4/19, 7/33)



(20/49, 29/71, 9/22, 11/27)
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(15/52, 17/59, 2/7, 13/45)
(9/56, 13/81, 4/25, 5/31)
(22/49, 31/69, 9/20, 13/29)



$(13 / 54,19 / 79,6 / 25,7 / 29)$

(21/50, 29/69, 8/19, 13/31)
(32/47, 47/69, 15/22, 17/25)

$(12 / 55,19 / 87,7 / 32,5 / 23)$
(20/51, 31/79, 11/28, 9/23)


$(16 / 53,19 / 63,3 / 10,13 / 43)$

(23/50, 29/63, 6/13, 17/37)
(26/49, 43/81, 17/32, 9/17)



$(30 / 49,41 / 67,11 / 18,19 / 31)$
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(27/50, 47/87, 20/37, 7/13)

(37/48, 47/61, 10/13, 27/35)

(32/49, 49/75, 17/26, 15/23)

(20/53, 23/61, 3/8, 17/45)

(11/58, 15/79, 4/21, 7/37)

(16/55, 23/79, 7/24, 9/31)

(23/52, 27/61, 4/9, 19/43)
(34/49, 59/85, 25/36, 9/13)

(36/49, 61/83, 25/34, 11/15)
(8/61, 13/99, 5/38, 3/23)

(31/50, 49/79, 18/29, 13/21)
(22/53, 27/65, 5/12, 17/41)


(13/58, 15/67, 2/9, 11/49)

(17/56, 27/89, 10/33, 7/23)

(24/53, 43/95, 19/42, 5/11)


(15/58, 23/89, 8/31, 7/27)



